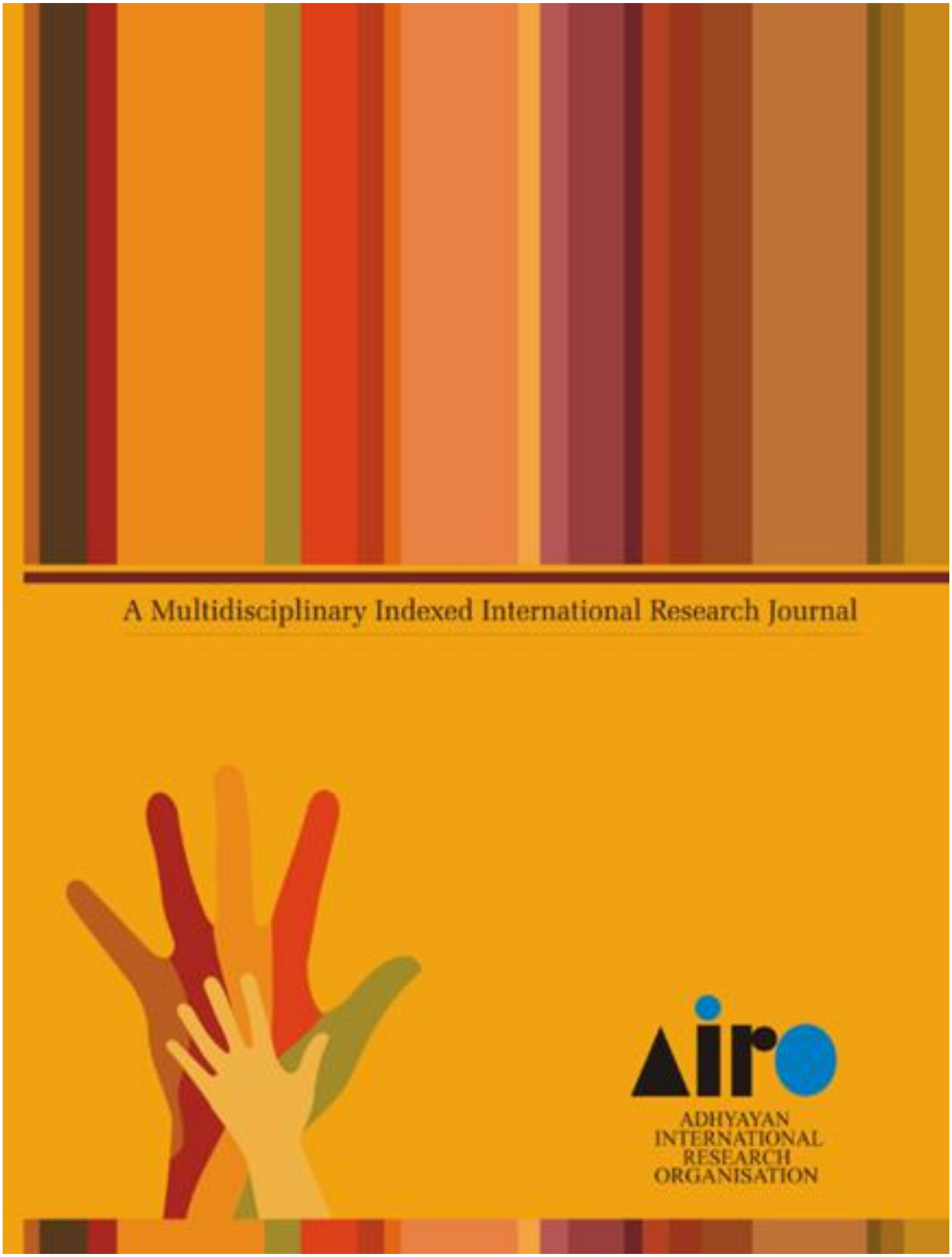
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# DIFFERENCE SEQUENCE SPACE WITH FUZZY METRIC SPACE

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***Abstract***

*In this paper we discuss the notion of difference operator* $\Delta_m (m \geq 0$ *an integer) for studying properties of some sequence space. Here we discuss the sequence spaces* $c^F(\Delta_m),\ c_0^F(\Delta_m),\ \ell_\infty^F(\Delta_m)$ *and study the completeness property of this sequence space.*

***Key words****: Fuzzy Sequence, difference sequence, Fuzzy metric space and Completeness*

The Concept of fuzzy set was introduced by Zadeh [19]. Bounded and convergent sequence of fuzzy numbers were studied by Matloka [20], where it is shown that every convergent sequence is bounded. Later on different classes of sequences of fuzzy numbers have been studied by Esi [1], Tripathy and Nanda [2], Savas [17], Fang and Hang [23], Choudhury and Tripathy [4] and many Others.

Let $D$ denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on real line $R$. For $X,\ Y \in D$, we define

$$d(X,Y) = \max\left(|a_1 - b_1|, |a_2 - b_2|\right) \text{ where } X = [a_1, a_2],\ Y = [b_1, b_2]$$

It is known that $(D, d)$ is a complete metric space.

A fuzzy real number $X$ is fuzzy set on $R$ and is a mapping $X : R \to I (= [0,1])$ associating each real number $t$ with its grade membership $X(t)$.

A sequence space $E$ is said to be normal (or solid) if $(Y_k) \in E$ , whenever $|Y_k| \leq |X_k|$ for all $k \in N$ and $(X_k) \in E$.

For a sequence $x = (x_k)$, $S(x)$ denote the set of all permutation of the elements of $(x_k)$ that is $S(x)=\{(x_{\pi(k)})\}$, where $\pi$ denote permutation over $N$.

A sequence spaces $E$ is said to be symmetric if $S(x)\subseteq E$, for all $x\in E$.

Let $K=\{k_1<k_2<k_3 \ .\ .\ .\ .\ \}\subseteq N$ and $E$ be a sequence space. A K-step space of $E$ is a sequence space $\lambda_K^E=\{(X_{k_n})\in w:(X_n)\in E\}$

A canonical pre-image of a step space $\lambda_K^E$ is a set of canonical pre-images of all elements in $\lambda_K^E$ i.e.,$Y$ is a canonical pre-image $\lambda_K^E$ if and only if $Y$ is a canonical pre-image of some $X\in\lambda_K^E$.

A sequence space $E$ is said to be monotone if it contains the canonical pre-images of all its step spaces.

From the above definition we have the following remark.

Remark 1. A sequence space $E$ is solid $\Rightarrow$ $E$ is monotone. It may refer to Kamthan and Gupta.

A sequence space $E$ is said to be a sequence algebra if $(x_k y_k)\in E$, whenever $(x_k),(y_k)\in E$.

A sequence space $E$ is said to be convergence free if $(y_k)\in E$ whenever $(x_k)\in E$ and $x_k=0$ implies $y_k=0$.

Fuzzy real numbers are considered as generalization of interval and multi-level interval numbers and so the knowledge of interval arithmetic is a basic requirement to introduce the fuzzy arithmetic. The following arithmetic operations will serve as basic knowledge for better understanding of the different operations of fuzzy number.

Consider the closed intervals $A = [a_1, a_2]$ and $B =[b_1, b_2]$

The $\alpha$ - level set of a fuzzy real number $X$, for $0<\alpha\leq 1$ denoted $X^\alpha$ is defined as $X^\alpha=\{t\in R: X(t)\geq\alpha\}$; for $\alpha$, it is the closure of the strong 0-cut (i.e. closure of the set $\{t\in R: X(t)>0\}$) . Throughout the article $\alpha$ means $\alpha\in(0,1]$ unless otherwise stated.

For the arithmetic operations on the closed intervals $A =[a_1, a_2]$ and $B = [b_1, b_2]$. The arithmetic operations on the closed intervals as follows

(i) $A+B=[a_1+b_1, a_2+b_2]$

(ii) $A-B=[a_1-b_2, b_1-a_2]$

(iii) $A\times B=[c_1, c_2]$

where $c_1=\min\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$

and $c_2=\max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$

(iv) $1\div A=\left[a_2^{-1}, a_1^{-1}\right]$, provided $0\notin A$.

Let $D$ denote the set of all closed and bounded intervals on $R$. The order relation between two intervals $A$ and $B$ as follows;

$$A\le B,\ \text{if}\ a_1\le b_1\ \text{and}\ a_2\le b_2$$

A fuzzy real number $X$ is called convex if $X(t)\ge X(s)\wedge X(r)=\min\,(X(s),X(r))$ where $s<t<r$.

If there exists $t_0\in R$ such that $X(t_0)=1$ then the fuzzy real number $X$ is called normal.

A fuzzy real number $X$ is said to be upper-semi continuous if, for each $\varepsilon>0$, $X^{-1}([0,a+\varepsilon])$, for all $a\in I$ is open in the usual topology of R, where $I=[0,1]$

The set of all upper semi continuous, normal, convex fuzzy real numbers is denoted by $R(I)$. Throughout the article, by a fuzzy real number we mean that the number belongs to $R(I)$.

The set $R$ of all real numbers can be embedded in $R(I)$. For $r\in R$, $\bar{r}\in R(I)$ is defined by

$$\bar{r}(t)=\begin{cases}1, & \text{for}\ t=r\\ 0, & \text{for}\ t\ne r\end{cases}$$

For $r\in R$ and $X\in R(I)$, the product $rX$ is defined

$$rX(t)=\begin{cases}X(r^{-1}t), & \text{for}\ r\ne 0\\ \bar{0}, & \text{for}\ r=0\end{cases}$$

The absolute value , $|X|$ of $X\in R(I)$ is defined by (use for instance Kaleva and Seikkala [25])

$$|X|(t)=\begin{cases}\max(X(t),X(-t)), & \text{for}\ t\ge 0\\ 0, & \text{for}\ t<0\end{cases}$$

A fuzzy real number is non negative if $X(t) = 0$, for all $t < 0$. The set of all non-negative fuzzy real number is denoted by $R^*(I)$.

Let $\overline{d} : R(I) \times R(I) \to R$ be defined by

$$\overline{d}(X,Y) = \sup_{0 \le \alpha \le 1} d(X^\alpha, Y^\alpha)$$

It is well known that $(R(I), \overline{d})$ is a complete metric space. Then $\overline{d}$ defines a metric on $R(I)$.

The arithmetic operations for $\alpha$ - level sets are defined as follows

Let $X, Y \in R(I)$ and the $\alpha$ - level sets be $[X]^\alpha = [a_1^\alpha, b_1^\alpha]$, $[Y]^\alpha = [a_2^\alpha, b_2^\alpha]$. Then

$$[X \oplus Y]^\alpha = [a_1^\alpha + a_2^\alpha, b_1^\alpha + b_2^\alpha],$$

$$[X - Y]^\alpha = [a_1^\alpha - b_2^\alpha, b_1^\alpha - a_2^\alpha],$$

$$[X \otimes Y]^\alpha = \left[\min_{i,j \in [1,2]} a_i^\alpha b_j^\alpha, \max_{i,j \in [1,2]} a_i^\alpha b_j^\alpha\right]$$

These operations can be defined other ways as follows

$$(X \oplus Y)(t) = \sup_{s \in R} \{X(s) \wedge Y(t-s)\}, \quad t \in R$$

$$(X - Y)(t) = \sup_{s \in R} \{X(s) \wedge Y(s-t)\}, \quad t \in R$$

$$(X \otimes Y)(t) = \sup_{s \in R, s \neq 0} \{X(s) \wedge Y(t/s)\}, \quad t \in R$$

$$(X \div Y)(t) = \sup_{s \in R, s \neq 0} \{X(ts) \wedge Y(s)\}, \quad t \in R$$

A fuzzy real valued sequence space is denoted by $(X_k)$ where $X_k \in R(I)$, for all $k \in N$ .

Throughout a sequence space , we mean a class of sequences of fuzzy real number is associated with a metric, which is a metric space.

A fuzzy real-valued sequences $(X_k)$ is said to be level convergent to the fuzzy real number $X$, if for each $\alpha \in (0,1]$

$$\lim_{k\to\infty} a_k^\alpha = a^\alpha$$

and $$\lim_{k\to\infty} b_k^\alpha = b^\alpha$$

where $[X_k]^\alpha = [a_k^\alpha, b_k^\alpha]$ and $[X]^\alpha = [a^\alpha, b^\alpha]$

If the convergence is uniform in $\alpha$, then we say that ($X_k$) converges uniformly to $X$.

A sequence ($X_k$) of fuzzy real number is said to be convergent (unifromly) to the fuzzy real number $X_0$ if , for $\varepsilon > 0$, there exist $n_0 \in N$ such that

$$\bar{d}(X_k, X_0) < \varepsilon, \quad \text{for all } k \geq n_0$$

A fuzzy real number ($X_k$) is said to bounded if there exist a $\mu \in R^*(I)$, the set of all non-negative fuzzy real numbers, such that $|X_k| \leq \mu,$ for all $k \in N$

DEFINITIONS AND PRELIMINARIES

After introduction of $R(I)$, different classes of fuzzy real valued sequences were introduced and studied by Tripathy [2] and others. Through the article $w^F$, $c^F$, $c_0^F$, $\ell_\infty^F$ denote the classes of all convergent, null and bounded sequences space of fuzzy real numbers.

A fuzzy real number $X$ is non negative if $X(t) = 0$ if for all $t < 0$, we will denote the set of all non negative fuzzy number by $G$.

DEFINITIONS. Let $X$ be any non empty set , $d : X \times X \to G$ and let the mapping $L, R : [0,1]\times[0,1] \to [0,1]$ be symmetric, non-decreasing in both arguments satisfy L(0, 0) =0 and R(1, 1) =1

Then

$$[d(x,y)]_\alpha = [\lambda_\alpha(x,y), \rho_\alpha(x,y)]$$

For all $x, y \in X$ and $0 < \alpha \leq 1$ and the quadruple ($X$, $d$, $L$, $R$) is called fuzzy metric space, if the following conditions are satisfied

(1) $d(x,y)=\bar{0}$ if and only if $x=y$

(2) $d(x,y)=d(y,x)$ for all $x,y\in X$

(3) For all $x,y,z\in X$

$(i)$ $d(x,y)(s+t)\geq L\left(d(x,z)(s),d(z,y)(t)\right)$

when $s\leq\lambda_1(x,z),\ t\leq(z,y)$ and $s+t\leq\lambda_1(x,y)$

$(ii)$ $d(x,y)(s+t)\leq R\left(d(x,z)(s),d(z,y)(t)\right)$

when $s\geq\lambda_1(x,z),\ t\geq\lambda_1(z,y)$ and $s+t\geq\lambda_1(x,y)$

The last condition is the triangle inequality. It is known that the triangle inequality (3)(ii) with *R*=max is equivalent to the triangle inequality

$$\rho_\alpha(x,y)\leq\rho_\alpha(x,z)+\rho_\alpha(z,y)$$

for $\alpha\in(0,1]$ and $x,y,z\in X$

Again the triangle inequality $3(i)$ with $L=\min$ is equivalent to the triangle inequality

$$\lambda_\alpha(x,y)\leq\lambda_\alpha(x,z)+\lambda_\alpha(z,y)$$

for $\alpha\in(0,1]$ and $x,y,z\in X$.

Throughout the chapter we shall consider the fuzzy metric space (*X*, *d*, min, max) unless otherwise stated.

It is known that in a fuzzy metric space (*X*, *d*, min, max) the triangle inequality (*iii*) is equivalent to

$$d(x,y)\leq d(x,z)+d(z,y)$$

Let (*X*, *d*, *L*, *R*) be a fuzzy metric space. A sequence $\{X_n\}$ is said to converge to $X$ denoted by

$$\lim_{n\to\infty}X_n=X$$

If and only if

$$\lim_{n\to\infty} d(X_n, X) = \bar{0}$$

$$\text{i.e } \lim_{n\to\infty} \rho_\alpha(X_n, X) = 0, \text{ for } \alpha \in (0,1]$$

A sequence $\{X_n\}$ is said to be Cauchy sequence if

$$\lim_{m,n\to\infty} d(X_m, X_n) = \bar{0}.$$

$$\text{i.e } \lim_{m,n\to\infty} \rho_\alpha(X_n, X) = 0, \text{ for } \alpha \in (0,1]$$

In the fuzzy metric space $(X, d, L, R)$ every convergent sequence is also a Cauchy sequence.

A fuzzy metric space $(X, d, L, R)$ with $\lim_{a\to 0+} R(a,a) = 0$ is said to be complete if every Cauchy sequence in $X$ converges.

Kizmaz [18] defined the difference sequence space for the crisp set. This concept was further generalized by Tripathy and Esi [6] as follows.

Let $m \geq 0$, be an integer then $Z_1(\Delta_m) = \{(x_k) \in w : (\Delta_m x_k) \in Z_1\}$ for $Z_1 = \ell_\infty, c$ and $c_0$. where $\Delta_m x_k = x_k - x_{k+m}$ for all $k \in N$. For m = 1, the spaces $\ell_\infty(\Delta), c(\Delta), c_0(\Delta)$ are studied by Kizmaz [18].

The idea of Kizmaz [18 ] was applied by Savas [17 ] for introducing the notion of difference sequences for fuzzy real numbers and study their different properties.

We study the following difference sequence of fuzzy real numbers of Tripathy & Esi [6] as follows :

Let $m \geq 0$, be an integer then,

$$Z(\Delta_m) = \{(x_k) \in w^F : (\Delta_m x_k) \in Z_1\}, \quad Z_1 = \ell_\infty^F, \ c^F \text{ and } c_0^F$$

## Main Result

Theorem : Let $d$ be a fuzzy metric space, then the classes of sequences $c^F(\Delta_m)$, $c_0^F(\Delta_m)$ and $\ell_\infty^F(\Delta_m)$ are the complete metric spaces by the metric

$$\rho(X,Y)=\sum_{k=1}^{m} d(X_k,Y_k)+\sup_k d(\Delta_m X_k,\Delta_m Y_k)\,.$$

Solution : Let ($X^i$) be any Cauchy sequence in $\ell_\infty^F(\Delta_m)$, where
$X^i=\left(X_k^i\right)=\left(X_1^i,X_2^i,X_3^i,X_4^i,X_5^i,\ \ldots\ldots\right)\in\ell_\infty^F(\Delta_m)$, for each $i\in N$.

$$\rho\left(X^i,X^j\right)=\sum_{k=1}^{m} d(X_k^i,X_k^j)+\sup_k d(\Delta_m X_k^i,\Delta_m X_k^j)\to\bar{0},\text{ as } i,j\to\infty$$

Hence $\sum_{k=1}^{m} d(X_k^i,X_k^j)\to\bar{0}$, as $i,j\to\infty$

$\Rightarrow d(X_k^i,X_k^j)\to\bar{0}$ as $i,j\to\infty$, for $k$ = 1, 2, 3, .......$m$

$\Rightarrow\left[\lambda_\alpha(X_k^i,X_k^j),\rho_\alpha(X_k^i,X_k^j)\right]\to\bar{0}_\alpha$, as $i,j\to\infty$, for $k$ = 1, 2, 3, .......$m$

$\Rightarrow\left[\min\left\{\left|X_{k,1}^{i,\alpha}-X_{k,1}^{j,\alpha}\right|,\left|X_{k,2}^{i,\alpha}-X_{k,2}^{j,\alpha}\right|\right\},\max\left\{\left|X_{k,1}^{i,\alpha}-X_{k,1}^{j,\alpha}\right|,\left|X_{k,2}^{i,\alpha}-X_{k,2}^{j,\alpha}\right|\right\}\right]\to[0,0]$

as $i,j\to\infty$, for $k$ = 1, 2, 3, .......$m$

Now,

$\max\left\{\left|X_{k,1}^{i,\alpha}-X_{k,1}^{j,\alpha}\right|,\left|X_{k,2}^{i,\alpha}-X_{k,2}^{j,\alpha}\right|\right\}\to 0$, as $i,j\to\infty$, for $k$ = 1, 2, 3, .......$m$

$\Rightarrow\left|X_{k,1}^{i,\alpha}-X_{k,1}^{j,\alpha}\right|\to 0$ and $\left|X_{k,2}^{i,\alpha}-X_{k,2}^{j,\alpha}\right|\to 0$ as $i,j\to\infty$, for $k$ = 1, 2, 3, .......$m$

$\Rightarrow\left(X_{k,1}^{j,\alpha}\right)$ and $\left(X_{k,2}^{j,\alpha}\right)$ are Cauchy sequence in $R$ for all $\alpha$ with $0<\alpha\le 1$ and

for $k$ = 1, 2, 3, .......$m$

$\Rightarrow\left(X_{k,1}^{j,\alpha}\right)$ and $\left(X_{k,2}^{j,\alpha}\right)$ are convergent sequence in $R$ for all $\alpha$ with $0<\alpha\le 1$

and for $k$ = 1, 2, 3, .......$m$. Since $R$ is complete.

Let $\lim_{j\to\infty} X_k^j=X_k$, say for $k$ = 1, 2, 3, .......$m$.

Again

$$d\left(\Delta_m X_k^i, \Delta_m X_k^j\right) \to \bar{0}, \text{ as } i, j \to \infty \text{ and for all } k \in N.$$

$$\Rightarrow \left[\lambda_\alpha\left(\Delta_m X_k^i, \Delta_m X_k^j\right), \rho_\alpha\left(\Delta_m X_k^i, \Delta_m X_k^j\right)\right] \to \bar{0}_\alpha \text{ as } i, j \to \infty \text{ and for all } k \in N.$$

$$\Rightarrow \left[\min\left\{\left|\Delta_m X_{k,1}^{i,\alpha} - \Delta_m X_{k,1}^{j,\alpha}\right|, \left|\Delta_m X_{k,2}^{i,\alpha} - \Delta_m X_{k,2}^{j,\alpha}\right|\right\}, \max\left\{\left|\Delta_m X_{k,1}^{i,\alpha} - \Delta_m X_{k,1}^{j,\alpha}\right| \right.\right.$$

$$\left.\left.\left|\Delta_m X_{k,2}^{i,\alpha} - \Delta_m X_{k,2}^{j,\alpha}\right|\right\}\right]$$

as $i, j \to \infty$ and for all $k \in N$.

Now

$$\max\left\{\left|\Delta_m X_{k,1}^{i,\alpha} - \Delta_m X_{k,1}^{j,\alpha}\right|, \left|\Delta_m X_{k,2}^{i,\alpha} - \Delta_m X_{k,2}^{j,\alpha}\right|\right\} \to 0, \text{ as } i, j \to \infty \text{ and for all } k \in N.$$

$$\Rightarrow \left|\Delta_m X_{k,1}^{i,\alpha} - \Delta_m X_{k,1}^{j,\alpha}\right| \to 0 \text{ and } \left|\Delta_m X_{k,2}^{i,\alpha} - \Delta_m X_{k,2}^{j,\alpha}\right| \to 0, \text{ as } i, j \to \infty \text{ and for all } k \in N.$$

$\Rightarrow \left(\Delta_m X_{k,1}^{j,\alpha}\right)$ and $\left(\Delta_m X_{k,2}^{j,\alpha}\right)$ are Cauchy sequence in $R$, for all $\alpha$ with $0 < \alpha \le 1$ and

for all $k \in N$

$\Rightarrow \left(\Delta_m X_{k,1}^{j,\alpha}\right)$ and $\left(\Delta_m X_{k,2}^{j,\alpha}\right)$ are convergent sequence in $R$, for all $\alpha$ with $0 < \alpha \le 1$

and for all $k \in N$,

Since $R$ is complete

Let $\lim_{j\to\infty} \Delta_m X_k^j = X_k$, exists for $k = 1, 2, 3, \ldots\ldots m$

So we have $\lim_{j\to\infty} \Delta_m X_k^j = X_k$, exist for each $k \in N$,

(By Principle of mathematical induction)

Thus

$$\lim_{j\to\infty}\sum_{k=1}^{m} d(X_k^i, X_k^j) = \sum_{k=1}^{m} d(X_k^i, X_k) \to 0 \text{ as } i \to \infty$$

and

$$\lim_{j\to\infty}\sum_{k=1}^{m} d(\Delta_m X_k^i, \Delta_m X_k^j) = d(\Delta_m X_k^i, \Delta_m X_k) \to 0 \text{ as } i \to \infty$$

This gives

$$\sup_k d(\Delta_m X_k^i, \Delta_m X_k) \to 0, \text{ as } i \to \infty .$$

Hence for $\varepsilon > 0$, there exist a positive integer $n_0$ such that

$d(X_k^i, X_k) < \varepsilon$ and $d(\Delta_m X_k^i, \Delta_m X_k) < \varepsilon$ for all $k \in N$ and for all $i, j \geq n_0$.

Thus we have

$$\sum_{k=1}^{m} d(X_k^i, X_k) + \sup_k d(\Delta_m X_k^i, \Delta_m X_k) < 2\varepsilon \text{ for all } k \in N \text{ and for all } i, j \geq n_0 .$$

$$\Rightarrow \rho(X^i, X) < 2\varepsilon, \text{ for all } k \in N \text{ and for all } i, j \geq n_0$$

Now for some fixed $i \geq n_0$

We have

$$\sup_k \rho(X_k, \bar{0}) \leq 2\varepsilon + M, \text{ where } M = \sup_k \rho(X_k^i, \bar{0})$$

Hence $X = (X_k) \in \ell_\infty^F(\Delta_m)$ is complete. Similarly we can show that $c^F(\Delta_m)$ and $c_0^F(\Delta_m)$ are also complete.

Conclusions : Following the notion of difference operator $\Delta_m$, introduced by Tripathy and Esi [6], the difference sequences $c^F(\Delta_m)$, $c_0^F(\Delta_m)$, $\ell_\infty^F(\Delta_m)$ of fuzzy numbers have been studied. It is shown that these classes of sequences are complete with fuzzy metric spaces.

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